

Why teaching functional programming to undergraduates at CUNY is important

Evan Misshula

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- 1 Why Functional Programming is intellectually interesting

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(particularly with Haskell)

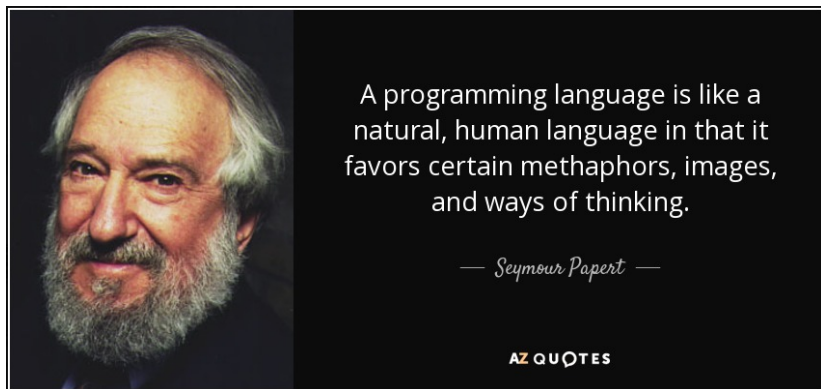
- ① Why Functional Programming is intellectually interesting
(particularly with Haskell)
 - ① The size and growth of the Tech sector in NYC
 - ② The size, growth and earnings of CUNY CS grads
 - ③ The demographic bias of the Tech Industry relative to NYC Population
 - ④ My thoughts on how helping to close this gap can benefit you and your employer

First computers were imperative by necessity

```
55 89 e5 53 83 ec 04 83 e4 f0 e8 31 00 00 00 89 c3 e8 2a 00
00 00 39 c3 74 10 8d b6 00 00 00 00 39 c3 7e 13 29 c3 39 c3
75 f6 89 1c 24 e8 6e 00 00 00 8b 5d fc c9 c3 29 d8 eb eb 90
```

Programming languages help us think





The Value of Programming Paradigms

- To be taught in universities
- To ignite flamewars
- To characterize programming languages
- To inspire memes



Confusing Syntax 2

```
1 A="Hello World"
2 if [ $A == $A ]; then
3     echo "Yes"
4 else
5     echo "No"
6 fi
```

- Outputs: "No"
- Is actually a syntax error!!
- \$A must be wrapped in double quotes

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- Outputs: "No"
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- \$A must be wrapped in double quotes

- actually the slide is wrong
- comparison should be [[

You can't talk about poor language design and not mention JS

```
console.log(0.1 + 0.2);  
console.log(0.1 + 0.2 == 0.3);
```

You can't talk about poor language design and not mention JS

```
console.log(0.1 + 0.2);  
console.log(0.1 + 0.2 == 0.3);
```

- 0.30000000000000004
- false

Comparisons can fail

```
console.log(1 < 2 < 3);  
console.log(3 > 2 > 1);
```

Comparisons can fail

```
console.log(1 < 2 < 3);  
console.log(3 > 2 > 1);
```

- true (1<2) -> true is implicitly coerced to 1 and 1<3
- false (3>2) -> true coerced to 1 and 1>1 is false

Even assignment is perilous

```
var a= [1,2,3];  
a[10]=99;  
console.log(a[10])  
console.log(a[6])
```


Even assignment is perilous

```
var a= [1,2,3];  
a[10]=99;  
console.log(a[10])  
console.log(a[6])
```

- 99
- [1, 2, 3, <7 empty items>, 99]

Yet Haskell allows us to

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- Rewrite classic sort algorithms in breathtakingly simple form

Yet Haskell allows us to

- Explore recursion both in functions and in data structures
- Rewrite classic sort algorithms in breathtakingly simple form
- Introduce students to algebraic ideas on functions so that they can master abstraction

Let's find a problem that puts constraints on tuples

- Which right triangle that has integers for all sides and all sides equal to or smaller than 10 has a perimeter of 24?

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- crack the problem like an egg

Right triangle problem

Let's find a problem that puts constraints on tuples

- Which right triangle that has integers for all sides and all sides equal to or smaller than 10 has a perimeter of 24?
- crack the problem like an egg
- Opportunity to teach: **solution by problem relaxation**

Right triangle problem relax solution

Integer sides all < 10 and perimeter = 24

- generate all tuples of sides less than 10

Right triangle problem relax solution

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- generate all tuples of sides less than 10
- designate z as the hypotenuse (bigger than x and y)

Right triangle problem relax solution

Integer sides all < 10 and perimeter = 24

- generate all tuples of sides less than 10
- designate z as the hypotenuse (bigger than x and y)
- make $x^2 + y^2 = z^2$

```
:set +m
length([(x,y,z) | x<-[1..10],y<-[1..10],
z<-[1..10],y<z,x<z,
(x^2 + y^2 == z^2)])
i==i
```

Prelude Control.Applicative | Prelude Control.Applicative | 4

Adding the perimeter constraint

Let's add constraints

- the perimeter equal 24
- $a + b + c = 24$

```
:set +m
```

```
length([(x,y,z) | x<-[1..10],y<-[1..10],z<-[1..10],  
y<z,  
x+y+z==24,  
(x^2 + y^2 == z^2)])  
[(x,y,z) | x<-[1..10],y<-[1..10],z<-[1..10],y<z,  
x+y+z==24,  
(x^2 + y^2 == z^2)]  
i==i
```

Haskell is statically typed

- Haskell allows students inquire about the type
 - We can see that type by using the `:t` command in the repl:

```
:t 'a'
```

```
:t True
```

```
:t "HELLO!"
```

```
:t (True, 'a')
```

```
:t 4 == 5
```

```
1==1
```

```
'a' :: Char
```

```
True :: Bool
```

```
"HELLO!" :: [Char]
```

```
(True, 'a') :: (Bool, Char)
```

```
4 == 5 :: Bool
```

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```
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```

```
:t "HELLO!"
```

```
:t (True, 'a')
```

```
:t 4 == 5
```

```
1==1
```

```
'a' :: Char
```

```
True :: Bool
```

```
"HELLO!" :: [Char]
```

```
(True, 'a') :: (Bool, Char)
```

```
4 == 5 :: Bool
```

Decompose the typeclass

```
(==) :: Eq a => a -> a -> Bool
```

- Typeclass constraint
 - The declaration we can read says:

Decompose the typeclass

$(==) :: \text{Eq } a \Rightarrow a \rightarrow a \rightarrow \text{Bool}$

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 - The declaration we can read says:
- The equality function takes two variables of the same type and returns a Bool

Decompose the typeclass

$(==) :: \text{Eq } a \Rightarrow a \rightarrow a \rightarrow \text{Bool}$

- Typeclass constraint
 - The declaration we can read says:
- The equality function takes two variables of the same type and returns a Bool
- The new part ' $\text{Eq } a \Rightarrow$ ' says:
- The type must be part of Eq typeclass
 - This is called the class constraint

The Eq typeclass provides an interface for testing for equality

- Eq is used for types that support equality testing
 - Its members implement both:
 - '=='
 - '/='

```
5==5
```

```
5/=5
```

```
'a' == 'a'
```

```
"Ho Ha" == "Ho Ha"
```

```
3.4 == 3.4
```

```
1==1
```

```
True
```

```
False
```

```
True
```

```
True
```

Introducing Ord typeclass

Ord is for types that have an ordering

- We can see the type of '>' comparison
- We can see some functions which rely on being in the ord typeclass

```
:t (>)
```

```
"Abc"< "Zev"
```

```
compare "Abc" "Zev"
```

```
5 >= 2
```

```
compare 5 3
```

```
1==1
```

```
(>) :: Ord a => a -> a -> Bool
```

```
True
```

```
LT
```

```
True
```

```
GT
```

Ord has a connection with inference

Ord is important in statistics

- Ord can be used to explain: **ordinal levels of measurement**

Ord has a connection with inference

Ord is important in statistics

- Ord can be used to explain: **ordinal levels of measurement**
- Ord can also be used to introduce: **utility curves**

Introducing Show typeclass

Everything except function has been part of show

- It works like Java or Ruby's toString methods
- Mostly we use it to examine a value

```
show 3
show 5.334
show True
1==1

3
5.334
True
```

Introducing Read typeclass

Read is the inverse of show

- It works reads a string and returns a type which supports the interface Read

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- You can use it to create Javascript like craziness

Introducing Read typeclass

Read is the inverse of show

- It works reads a string and returns a type which supports the interface Read
- You can use it to create Javascript like craziness
- **But you have to work at it**

```
read "True" || False
read "8.2" + 3.8
read "5" - 2
read "[1,2,3,4]" ++ [3]
1==1
```

```
True
12.0
3
```

Let's look at a type error

```
read 4
```

```
1==1
```

```
<interactive>:2327:6: error:
```

- Could not deduce (Num String) arising from the literal '4' from the context: Read a
bound by the inferred type of it :: Read a => a
at <interactive>:2327:1-6
- In the first argument of 'read', namely '4'
In the expression: read 4
In an equation for 'it': it = read 4
- GHCi is saying it does not know what type to return
 - Do you want an Float or an Integer?

We can specify a type

- We just add '::<Type>' and read will work

```
read "5" :: Int
read "5" :: Float
(read "5" :: Int) * 4
read "[1,2,3,4]" :: [Int]
read "(3,'a')" :: (Int, Char)
1==1
```

5

5.0

20

[1,2,3,4]

(3,'a')

Sequentially ordered types

- Being *sequentially ordered* means that they can be counted in order
- This property is also called being *enumerable*
- We can use them in list ranges
 - they each have a predecessor which you can get with 'pred'
 - they each have a successor which you can get with 'succ'

```
['a'..'e']
```

```
[LT .. GT]
```

```
[3..7]
```

```
succ 'B'
```

```
1==1
```

```
abcde
```

```
[LT,EQ,GT]
```

```
[3,4,5,6,7]
```

Bounded type class has concrete types

- with maximum and minimum elements
 - minBound and maxBound are functions with polymorphic type
 - (Bounded a) => a

```
minBound :: Int
```

```
maxBound :: Char
```

```
maxBound :: Bool
```

```
minBound :: Bool
```

```
i==i
```

```
-9223372036854775808
```

```
'\1114111'
```

```
True
```

```
False
```

Numeric types can be operated on mathematically

- Let's look at this type

```
:t (*)
```

```
(5 :: Int) * (6 :: Integer)
```

```
(5 :: Int) * 6
```

```
i==i
```

```
(*) :: Num a => a -> a -> a
```

```
<interactive>:2350:15: error:
```

- Couldn't match expected type 'Int' with actual type 'Integer'
- In the second argument of '(*)', namely '(6 :: Integer)'
In the expression: (5 :: Int) * (6 :: Integer)
In an equation for 'it': it = (5 :: Int) * (6 :: Integer)

30

Integral and Floating types

- The Integral typeclass only includes Integer and Int
- The Floating typeclass only includes floats and double

```
:t fromIntegral
```

```
fromIntegral (length [1,2,3,4]) + 3.2
```

```
i==i
```

```
fromIntegral :: (Num b, Integral a) => a -> b
```

```
7.2
```

Every function in haskell only takes one argument

- But what about 'max' or min?
 - We actually apply parameters to functions one at time
 - These are called "curried" functions
 - This is after Haskell Curry
- ```
max (Ord a) => a -> a -> a
max (Ord a) => a -> (a -> a)
```
- If we call a function with to few parameters we get back a partially applied function

```
:set +m
```

```
-- multThree :: (Num a) => a -> a -> a -> a
multThree x y z = x * y * z
multThree 3 5 9 == ((multThree 3) 5) 9
i==1
```



## Here is a curried comparison

- These are the same because 'x' is on both sides of the equation

```
-- compareWithHundred :: (Num a, Ord a, Show a) => a -> Ordering
compareWithHundred x = compare 100 x
```

```
-- compareWithHundred1 :: (Num a, Ord a, Show a) => a -> Ordering
compareWithHundred1 = compare 100
```

# Example partial application

## Let's look at an infix function

- simply surround the function with parentheses and only supply one of the parameters
- this is called 'sectioning'

```
-- divideByTen :: (Floating a) => a -> a
divideByTen = (/10)
```

## String functions can be partially applied too

- this is written in point free style
- it is also sectioned

```
-- isUpperAlphanum :: Char -> Bool
isUpperAlphanum = ('elem' ['A'..'Z'])
```

## Functions can return functions

- take a function and apply it twice

```
-- applyTwice :: (a -> a) -> a -> a
applyTwice f x = f (f x)
```

## We are going to implement ZipWith

- It joins two lists and performs a function on the corresponding elements

```
-- zipWith' :: (a -> b -> c) -> [a] -> [b] -> [c]
zipWith' _ [] _ = []
zipWith' _ _ [] = []
zipWith' f (x:xs) (y:ys) = f x y : zipWith' f xs ys
```

## flip changes the order of the arguments

```
-- flip' :: (a -> b -> c) -> (b -> a -> c)
flip' f = g
 where g x y = f y x

-- flip'' :: (a -> b -> c) -> b -> a -> c
flip'' f y x = f x y
```

## Map

- map takes a function applies the function to each element of a list

```
-- map :: (a -> b) -> [a] -> [b]
map _ [] = []
map f (x:xs) = f x : map f xs
```

## Filter

- 'filter' take a function called a predicate and a list of any type
- the predicate takes an element of the list and returns a Bool
  - the filter returns elements for which the predicate is True

```
filter :: (a -> Bool) -> [a] -> [a]
filter _ [] = []
filter p (x:xs)
 | p x = x : filter p xs
 | otherwise = filter p xs
```



## Lambdas are anonymous functions

- These are unnamed functions
- They are passed as parameters to other functions
- They work like composition in math
- They are called 'lambdas' because of the 'lambda calculus'



Alan Turing  
(1912 – 1954)



Alonzo Church  
(1903-1995)

Turing Machine

Lambda calculus

Two mathematical ways to ask questions about  
“computability”

## Lambda Calculus is a formal system for computation

- it is equivalent to calculation by Turing Machine
- invented by Alonzo Church in the 1930's
- Church was Turing's thesis advisor
  - a function is denoted by the greek letter  $\lambda$
  - a function  $f(x)$  that maps  $x \rightarrow f(x)$  is:
    - $\lambda x.y$

# Example of a lambda

## We can pass a lambda to ZipWith

- a lambda function in Haskell starts with ' $\backslash$ '
- can't define several parameters for one parameter

```
zipWith (\a b -> (a * 30 + 3) / b) [5,4,3,2,1] [1,2,3,4,5]
1==1
```

## specification

```
-- quicksort :: (Ord a) => [a] -> [a]
quicksort [] = []
quicksort (x:xs) =
 let smallerSorted = quicksort [a | a <- xs, a <= x]
 biggerSorted = quicksort [a | a <- xs, a > x]
 in smallerSorted ++ [x] ++ biggerSorted
1==1
```

## Folds encapsulate several functions with $(x:xs)$ patterns

- they reduce a list to a single value
- 'foldl' is the left fold function

```
sum' :: (Num a) => [a] -> a
sum' xs = foldl (\acc x -> acc + x) 0 xs
```

```
sum'' :: (Num a) => [a] -> a
sum'' = foldl (+) 0
1==1
```

## Function application with \$

- '\$' is called the *function application*
- changes to right association
- keeps us from writing parentheses

```
map ($ 3) [(4+), (10*), (^2), sqrt]
1==1
```

```
[7.0,30.0,9.0,1.7320508075688772]** Exception: <interactive>:23
```

## Function composition is just like math

- In math  $f \cdot g(x) = f(g(x))$
- Let's look at Haskell function
- $g$  takes  $a \rightarrow b$
- $f$  takes  $b \rightarrow c$



## Function composition is just like math

- In math  $f \cdot g(x) = f(g(x))$
- Let's look at Haskell function
- $g$  takes  $a \rightarrow b$
- $f$  takes  $b \rightarrow c$
  
- so the composition take  $f \cdot g$  takes  $a \rightarrow c$

```
(.) :: (b -> c) -> (a -> b) -> a -> c
f . g = \x -> f (g x)
```

## Function composition examples

- with a  $\lambda$
- with point free notation

```
map (\x -> negate (abs x)) [5,-3,-6,7,-3,2,-19,24]
```

```
map (negate . abs) [5,-3,-6,7,-3,2,-19,24]
```

```
[-5,-3,-6,-7,-3,-2,-19,-24]** Exception: <interactive>:2394:1-2
```

# Polymorphism on a higher level

- Types are not part of a hierarchy
- We can think about how they should act
  - then connect them with typeclasses

## Definition (definition)

A functor is a typeclass for all the things that can be mapped over

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A functor is a typeclass for all the things that can be mapped over

## Definition (Haskell syntax definition)

- class Functor f where  
    fmap (a -> b) -> f a  
        -> f b

## Typeclasses define functions

- Eq *defines* concrete types that are equatable
  - functions ('=') and ('/')
- Ord *defines* concrete types that 'orderable'
  - implements the 'compare' function
- Enum *defines* concrete types that enumerable
  - defines '..' a range

## Example (List Functor Examples)

- $\text{map} :: (a \rightarrow b) \rightarrow [a] \rightarrow [b]$
- `instance Functor []` where
  - $\text{fmap} = \text{map}$

# Functor code in the repl

## List Functor in the repl

```
:t map
fmap (*2) [1..3]
map (*2) [1..3]
i==i

map :: (t -> a) -> [t] -> [a]
[2,4,6]
[2,4,6*** Exception: <interactive>:2394:1-29: Non-exhaustive pa
```



## Example (Maybe Functor Examples)

```
type myMaybe a = Nothing | Just a
```

```
instance Functor myMaybe where
```

```
 fmap f (Just x) = Just (f x)
```

```
 fmap f Nothing = Nothing
```

# Maybe Functor code in the repl

## Maybe Functor in the repl

```
:t fmap
```

```
fmap (++) " HEY GUYS IM INSIDE THE JUST") (Just "Something serious")
```

```
fmap (++) " HEY GUYS IM INSIDE THE JUST") Nothing
```

```
fmap (*2) (Just 200)
```

```
fmap (*2) Nothing
```

```
i==i
```

```
fmap :: Functor f => (a -> b) -> f a -> f b
```

```
Just "Something serious. HEY GUYS IM INSIDE THE JUST"
```

```
Nothing
```

```
Just 400
```

```
Nothing
```

If functors mean that something can be mapped over...

- then calling 'fmap' on a functor should
  - map a function over the functor

If functors mean that something can be mapped over...

- then calling 'fmap' on a functor should
  - map a function over the functor
- **Nothing else**

## Definition (The First Functor Law)

states that if we map the identity (`id`) function over a functor, we get the functor

- $\text{fmap id} = \text{id}$

# Identity in the Repl

## Identity functions in the repl

```
fmap id (Just 3)
```

```
id (Just 3)
```

```
fmap id [1..5]
```

```
id [1..5]
```

```
fmap id []
```

```
fmap id Nothing
```

```
1==1
```

```
Just 3
```

```
Just 3
```

```
[1,2,3,4,5]
```

```
[1,2,3,4,5]
```

```
[]
```

```
Nothing
```

## Definition (The Second Functor Law says)

The Second Functor Law says that composing two functions and then mapping the composed function over a functor is the same as first mapping one function over the functor and then mapping the other one.

- $\text{fmap } (f.g) = \text{fmap } f . \text{fmap } g$
- $\text{fmap } (f.g) F = \text{fmap } f (\text{fmap } g F)$

## Composition functions in the repl

```
fmap ((+1).(*2)) (Just 3)
fmap (+1) (fmap (*2) (Just 3))
fmap ((+1).(*2)) [1..5]
fmap (+1) (fmap (*2) [1..5])
1==1
```

```
Just 7
```

```
Just 7
```

```
[3,5,7,9,11]
```

```
[3,5,7,9,11]
```



# What if we map a multi-parameter function over a functor?

- Look at the type signature

```
a = fmap (*) [1..4]
:t a
fmap (\f -> f 9) a
1==1
```

```
a :: (Num a, Enum a) => [a -> a]
[9,18,27,36]
```

# What if we want to take a function out of a Just

Let's take a Just (3 \*) and map

and map it over Just 5

```
:set +m
:{
class (Functor f) => Applicative f where
 pure :: a -> f a;
 (<*>) :: f (a -> b) -> f a -> f b
:}
```

## Let's look at the Applicative for Maybe

```
:set +m
:{
instance Applicative MyMaybe where
 pure = Just
 Nothing <*> _ = Nothing
 (Just f) <*> something = fmap f something
:}
```

# Maybe Applicative inside the repl

## Using the Maybe Applicative

```
-- :add Control.Applicative
Just (+3) <*> Just 9
pure (*2) <*> Just 10
pure (+3) <*> Just 9
Just (++"!!") <*> Just "Go now"
Nothing <*> Just "woot"
1==1
```

```
<interactive>:2476:1: error:
```

- Could not deduce (Applicative Maybe) arising from a use of <\*> from the context: Num b  
bound by the inferred type of it :: Num b => Maybe b  
at <interactive>:2476:1-20
- In the expression: Just (+ 3) <\*> Just 9  
In an equation for 'it': it = Just (+ 3) <\*> Just 9

# Fmap as an infix operator

Control.Applicative exports a function called `<$>`

which is `fmap` as an infix operator

```
(<$>) :: (Functor f) => (a->b) -> f a -> f b
```

```
f <$> x = fmap f x
```

# Compare Applicatives in the repl

## Infix fmap in the repl

```
(++) <$> Just "John " <*> Just "Travolta"
```

```
(++) "John " "Travolta"
```

```
1==1
```

```
<interactive>:2486:1: error:
```

- No instance for (Applicative Maybe) arising from a use of `(<\*>)`
- In the expression: (++) <\$> Just "John " <\*> Just "Travolta"
- In an equation for ‘it’:

```
 it = (++) <$> Just "John " <*> Just "Travolta"
```

```
John Travolta
```

## Definition (Definition of the Applicative for a list)

- Literally a Cartesian product of functions and list values

```
:set +m
: {
instance Applicative [] where
 pure x = [x]
 fs <*> xs = [f x | f <- fs, x <- xs]
: }
```

## Applicative Functors of lists in the repl

```
[(*0), (+100), (^2)] <*> [1..4]
```

```
[(+), (*)] <*> [1,2] <*> [3,4]
```

```
(++) <$> ["ha", "heh", "hmm"] <*> ["?", "!", "."]
```

```
1==1
```

```
[0,0,0,0,101,102,103,104,1,4,9,16]
```

```
[4,5,5,6,3,4,6,8]
```

```
["ha?", "ha!", "ha.", "heh?", "heh!", "heh.", "hmm?", "hmm!", "hmm."]
```



# IO is an Applicative

Let's see how the IO Applicative is implemented:

```
:set +m
:{
instance Applicative IO where
 pure = return
 a <*> b = do
f <- a
x <- b
return (f x)
:}
```

# Concatenating IO strings

## Two ways to concatenate two lines of user input string

- Imperative code

```
:set +m
:{
myAction :: IO String
myAction = do
 a <- getLine
 b <- getLine
 return $ a ++ b
:}
```

## Applicative way to concatenate two lines of user input string

- Applicative code

```
:set +m
:{
myAction :: IO String
myAction = (++)
 <$> getLine
 <*> getLine
:}
```

# The first Applicative Functor Law

## Theorem (The first Applicative Functor Law)

- $\text{pure } f \langle * \rangle x = \text{fmap } f x$

# Some lessons we've skipped

## Defining types

- *data* will define a new algebraic type
- *type* creates a type synonym
- *newtype* creates new types from old types

# Applicative Functor in two ways

## function left, each argument right

```
:m Control.Applicative
[(+1),(*100),(*5)] <*> [1..3]
1==1
```

```
[2,3,4,100,200,300,5,10,15]
```

## function left, every argument right

```
:set +m
:{
instance Applicative ZipList where
pure x = ZipList (repeat x)
ZipList fs <*> ZipList xs = ZipList
:}
 getZipList $ ZipList [(+1),(*
 -- getZipList $
 -- ZipList [(+1),(*100),(*5)]
 -- <*> ZipList [1,2,3]
1==1
```

```
Prelude Control.Applicative| Pr
```

# The newtype keyword

'newtype' takes one type and wrap it

- to present it as another type

```
newtype ZipList a = ZipList {getZipList :: [a]}
```

- data can have multiple value constructors

## 'data' to make new types

- Here are additive and multiplicative types with multiple constructors

```
data Profession = Fighter | Archer | Wizard
data Species = Human | Elf | Orc | Goblin
data PlayerCharacter = PlayerCharacter Species Profession
```

# Using newtype to drive typeclass properties

## newtype

```
newtype CharList = CharList {getCharList :: [Char]} deriving(Eq)
```

```
CharList "this will be shown!"
```

```
CharList "benny" == CharList "benny"
```

```
CharList "benny" == CharList "oysters"
```

```
1==1
```

```
CharList {getCharList = "this will be shown!"}
```

```
True
```

```
False
```



## Definition (Monoid definition)

A data type, category or set is a **monoid** if it has a binary operation  $\bullet$  which is associative and has an identity.

- $\forall a, b, c \in S, (a \bullet b) \bullet c = a \bullet (b \bullet c)$
- $e \bullet a = a \bullet e = a$

```
:set +m
:{
class Monoid m where
 mempty :: m
 mappend :: m -> m -> m
 mconcat :: [m] -> m
 mconcat = foldr mappend mempty
:}
```

## Defining the monoid functions

- 'mempty' is just the identity function
- mappend is the binary function
  - it doesn't just append
- mconcat reduces a list of monoid values and reduces them to one by applying mappend

## Theorem (The Monoid Laws are just the definition in Haskell)

- $mappend\ mempty\ x = x$
- $mappend\ x\ mempty = x$
- $mappend\ (mappend\ x\ y)\ z = mappend\ x\ (mappend\ y\ z)$

## Example (List is a monoid)

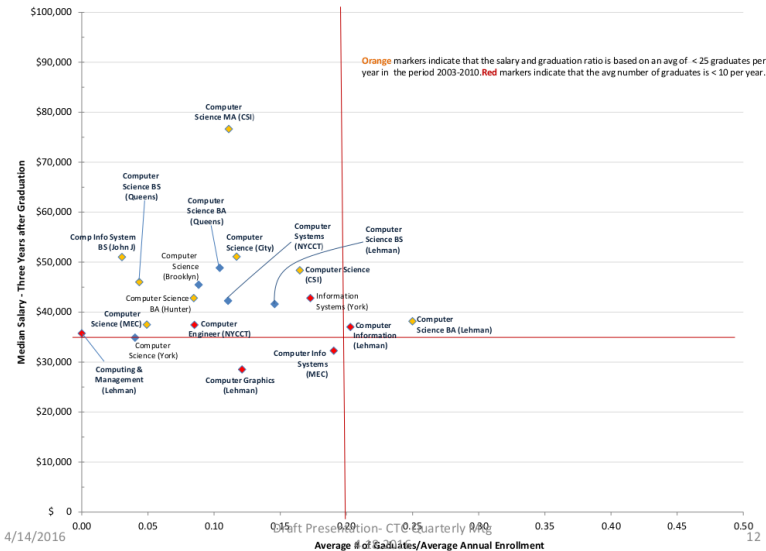
- `[]` with `(++)` is a monoid
  - `id = ""`
- Natural numbers with `(*)` is a monoid
  - `id = 1`
- Natural numbers with `(+)` is a monoid
  - `id = 0`

# Why is all of this important to you

- BLS Statistics
- 2015 median salary is \$100,690
- Number of jobs: 1,114,000
- Job growth: 17% (much faster than average)

- The Technology Sector in New York City 4/2018
- New York State had the third-largest tech sector in the nation in 2016.
- Employment in NYC's tech sector increased by 57% between 2010 and 2016 (46,900 jobs), 3x faster than the rest of the private sector
- The average salary increased 3x faster than the rest of the City's private sector to reach a record \$147,300 by 2016

## Estimated Median Annual Salary & Graduation Ratio by Academic Major Computer Science-Related Majors in Baccalaureate Degree Programs



4/14/2016

12

*In New York City, 44.6% of the population is white, 25.1% is black, and 11.8% are of Asian descent. Hispanics of any race represent about 27.5% percent of New York City's population*

- US Census 2018

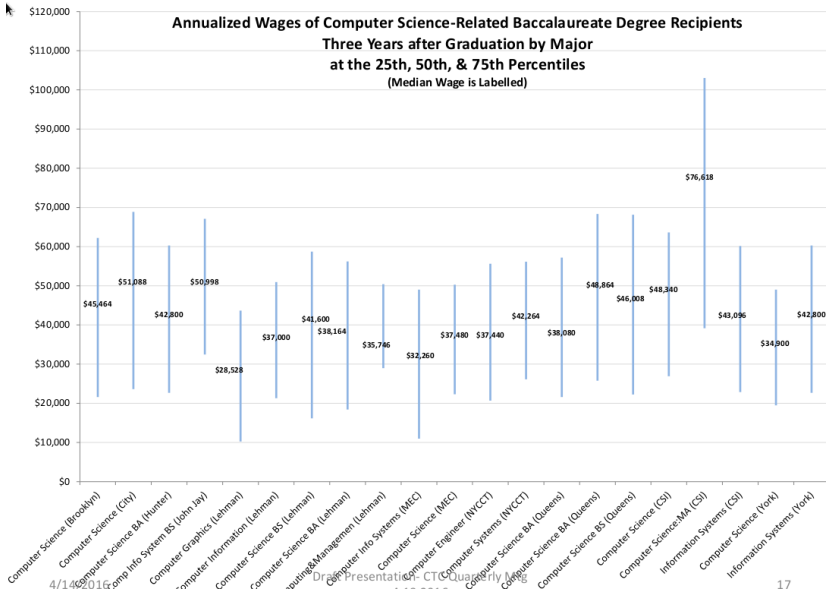


# NYC Tech Sector does not reflect our diversity

*[B]lacks and Latinos constitute 25.1 percent and 27.5 percent of the population, respectively, but only 9 percent and 11 percent, respectively, are employed in the tech sector.*

- City Limits: Why is NYC Tech so White?

## Annualized Wages of Computer Science-Related Baccalaureate Degree Recipients Three Years after Graduation by Major at the 25th, 50th, & 75th Percentiles (Median Wage is Labelled)



4/14/2016

17

- CUNY Student Experience 2016
- NYC Tech is 62% White, 60% male
- NYC Tech Profile
- Why NYC's Growing Tech Sector is so White
- Numbers say New York's tech boom is real
- Will Silicon Alley Be the Next Silicon Valley?
- The Technology Sector in New York City
- NYC Population
- CUNY 2x Initiative
- Peter Drake's Prog Lang Course Materials
- <http://learnyouahaskell.com/>